

Chapter 10: Diagnose vertical motion based on Q -Vectors

In the ω equation, terms B and C are associated with certain physical processes. In practice, when the ω equation is applied to diagnose the vertical motion, there is often a significant cancellation between these 2 terms. In addition, they are not invariant under a Galilean transformation of the zonal coordinates. Thus, an alternative approach, the Q -vector approach has been developed.

On the midlatitude β -plane the quasi-geostrophic prediction equations may be expressed simply as

$$\frac{D_g u_g}{Dt} - f_0 v_a - \beta y v_g = 0 \quad (6.38)$$

$$\frac{D_g v_g}{Dt} + f_0 u_a + \beta y u_g = 0 \quad (6.39)$$

$$\frac{D_g T}{Dt} - \frac{\sigma p}{R} \omega = \frac{J}{c_p} \quad (6.40)$$

These are coupled by the thermal wind relationship

$$f_0 \frac{\partial u_g}{\partial p} = \frac{R}{p} \frac{\partial T}{\partial y}, \quad f_0 \frac{\partial v_g}{\partial p} = -\frac{R}{p} \frac{\partial T}{\partial x} \quad (6.41a,b)$$

or in vector form:

$$\left(f_0 \mathbf{k} \times \frac{\partial \mathbf{V}_g}{\partial p} \right) = \frac{R}{p} \nabla T \quad (6.42)$$

Equations (6.38) – (6.40) with the usage of (6.41a, b) may be manipulated to obtain

$$\sigma \nabla^2 \omega + f_o^2 \frac{\partial^2 \omega}{\partial p^2} = -2 \nabla \cdot \mathbf{Q} \quad (6.45)$$

where \mathbf{Q} is defined as

$$\mathbf{Q} = -\frac{R}{p} \left| \frac{\partial T}{\partial y} \right| \mathbf{k} \times \frac{\partial \mathbf{V}_g}{\partial x}. \quad (6.55)$$

Since the left side of the above equation is a three-dimensional Laplacian, a simple form of (6.45) for determining the sign (upward or downward) of the vertical motion can be written as

$$\sigma \nabla^2 \omega + f_o^2 \frac{\partial^2 \omega}{\partial p^2} \propto -\omega \propto w \propto -\nabla \cdot \mathbf{Q} \quad (6.46)$$

or

$$w \propto -\nabla \cdot \mathbf{Q}. \quad (6.47)$$

In other words, in a region with convergence of \mathbf{Q} vectors ($\nabla \cdot \mathbf{Q} < 0$) will produce an upward motion ($w > 0$), and a region with divergence of \mathbf{Q} vectors will produce a downward motion.

The direction and magnitude of the \mathbf{Q} vector at a given point on a weather map can be estimated by referring the motion to a Cartesian coordinate system in which the x axis is parallel to the local isotherm with cold air on the left. Then (6.51) can be simplified to give

$$\mathbf{Q} = -\frac{R}{p} \left(\frac{\partial T}{\partial y} \right) \left(\frac{\partial v_g}{\partial x} \mathbf{i} - \frac{\partial u_g}{\partial x} \mathbf{j} \right)$$

where we have again used the fact that $\partial u_g / \partial x = -\partial v_g / \partial y$. From the rules for cross multiplication of unit vectors, the above expression for \mathbf{Q} can be rewritten as

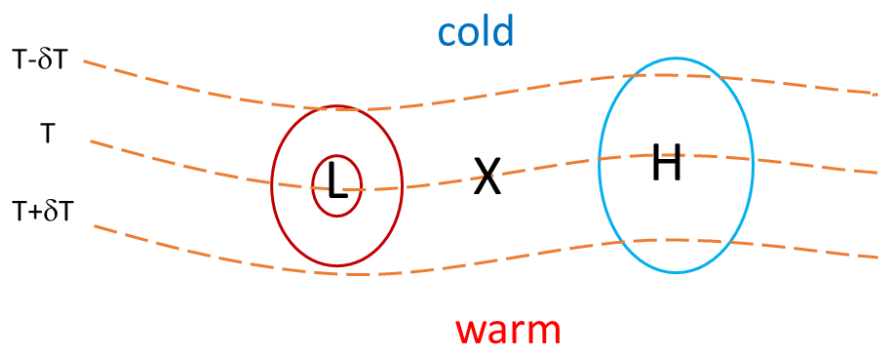
$$\mathbf{Q} = -\frac{R}{p} \left| \frac{\partial T}{\partial y} \right| \left(\mathbf{k} \times \frac{\partial \mathbf{V}_g}{\partial x} \right) \quad (6.55)$$

Thus, the \mathbf{Q} vector can be obtained by evaluating the vectorial change of \mathbf{V}_g along the isotherm (with cold air on the left), rotating this change vector by 90° clockwise, and multiplying the resulting vector by $|\partial T / \partial y|$.

In-class lab:

Use the handout lab and based on (6.46) or (6.47) and (6.55), determine the vertical motion at point A by the following the procedure listed below:

- (1) Define the x-axis//local isotherm leaving cold air to the left
- (2) Determine the Q vectors at the high (H) and low (L) centers by
 - (i) Find $\partial V_g / \partial x$ from 2 adjacent points (denoted by “x”) near H and L centers along the isotherms. You may assume V_g is along the isobars.
 - (ii) Rotating the $\partial V_g / \partial x$ clockwise by 90° to get Q vectors at H and L.
- (3) Determine the vertical velocity at X by $w \propto -\nabla \cdot Q$.



Homework: Determine the vertical motion at the entrance region of a jet streak.