ASME 434 **Atmospheric Dynamics II** **Dr. Yuh-Lang Lin**
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Chapter 13 Introduction to Planetary Boundary Layer

13.1 Reynolds Averaging

- Integrating the governing differential equations in a limited area numerically will limit the explicit representation of atmospheric motions and processes at a scale smaller than the grid interval, truncated wavelength, or finite element.

- The subgrid-scale disturbances may be inappropriately represented by the grid point values, which may cause nonlinear aliasing and nonlinear numerical instability.

- One way to resolve the problem is to explicitly simulate any significant small-scale motions and processes. This is called *direct numerical simulation* (*DNS*) or *full turbulence simulation* (*FTS*).

- In DNS or FTS, the time-dependent *Navier-Stokes equations* with explicit terms for molecular diffusion are integrated numerically. This requires a grid interval to be finer than the smallest scales of motion in the solution.

- The DNS is limited to moderate *Re* ($Re = UL/\nu$) number flows (Mason 1994).

Re~10^3 to 10^6 in engineering practice
Re~10^6-10^9 in atmospheric boundary layer
(see Arya 1988)

- It is unrealistic to apply DNS to mesoscale modeling, not to mention NWP modeling.

- The second approach is to simulate *large turbulent eddies* explicitly. This is called *large-eddy simulations* (*LES*) (Deardorff 1970; Mason 1994).

- Both DNS and LES produce highly fluctuating variables in time and space. This leads to a third way that numerically integrates the *Reynolds-averaged equations* of the mean motion.

  The ensemble properties of all time fluctuations in the flow are described by a turbulence closure. In this approach, the subgrid-scale motions and processes as well as the larger-scale environment is *parameterized*.

- Following the scheme originally developed by Reynolds (1895), each model variable may be decomposed into a slow-varying part and a rapid fluctuating part, such as

  $u = \overline{u} + u'$, $v = \overline{v} + v'$, $w = \overline{w} + w'$, $\theta = \overline{\theta} + \theta'$, $p = \overline{p} + p'$, $\rho = \overline{\rho} + \rho'$, etc.

Some useful formulas for the *Reynolds averaging* may be derived, for example,

$$\overline{u+w}=\bar{u}+\bar{w};\quad \overline{cw}=c\bar{w};\quad \bar{\bar{w}}=\bar{w};\quad \overline{w'}=0; \qquad (14.1.1)$$

$$\overline{w'\bar{\theta}}=\overline{w'}\,\bar{\theta}=0;\quad \overline{w\theta}=\overline{(\bar{w}+w')(\bar{\theta}+\theta')}=\bar{w}\,\bar{\theta}+\overline{w'\theta'}\;;\quad \overline{uw}=\bar{u}\,\bar{w}+\overline{u'w'};$$

$$\overline{\frac{\partial w}{\partial s}}=\frac{\partial \bar{w}}{\partial s},\qquad \overline{\frac{\partial \bar{w}}{\partial s}}=\frac{\partial \bar{w}}{\partial s},\qquad \int\overline{w\,ds}=\int\bar{w}\,ds,\qquad s=x,y,z,\text{or } t.$$

where $\overline{u'w'}$ and $\overline{w'\theta'}$ are called a *vertical turbulent flux of horizontal momentum* and a *vertical turbulent heat flux*, respectively.

- In statistical terms, these fluxes, as an average of the product of deviation components, are also called *covariances*.

Fig. 14.1.1 shows the subgrid scale covariance $\overline{w'\theta'}$.

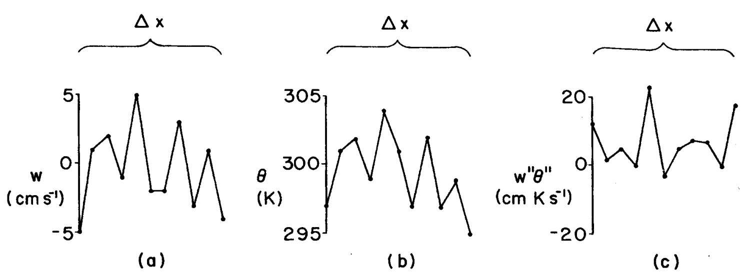

Fig. 14.1.1: Schematic illustration of subgrid scale values of vertical velocity *w*, potential temperature *θ*, and the subgrid scale correlation *w'θ'*. In this example, the grid averaged value of vertical motion is required to be approximately 0 (i.e. $\bar{w}=0$, and $\bar{\theta}=299.5K$ is used. Both grid value averages are assumed to be constant over Δx. The grid-averaged subgrid-scale correlation $\overline{w'\theta'}$ is equal to $6.9\,cm\,K\,s^{-1}$. (Adapted from Pielke 2002)

- In this example, the grid-averaged value of the vertical velocity is approximately zero, $\overline{w'} = 0$, and $\overline{\theta'} = 0$. Both grid-averaged values are assumed to be constant over the grid interval, Δx.

  However, the covariance or the vertical turbulent heat flux, $\overline{w'\theta'}$, is not 0.

- If we apply the Reynolds averaging to a grid volume of a numerical model, then the Reynolds-averaged value of a variable ϕ represents,

$$\overline{\phi} \equiv \frac{1}{\Delta x\, \Delta y\, \Delta z\, \Delta t} \int_t^{t+\Delta t} \int_x^{x+\Delta x} \int_y^{y+\Delta y} \int_z^{z+\Delta z} \phi \; dz\, dy\, dx\, dt \,. \qquad (14.1.2)$$

  This is called *grid-volume averaging*. Thus, ϕ' is the fluctuation or perturbation across the grid intervals, $\Delta x, \Delta y, \Delta z$, and time interval Δt from $\overline{\phi}$.

- Applying the Reynolds averaging to the grid volume of the mesoscale model system of Eqs. (15.5.6)-(15.5.10) with anelastic approximation leads to (e.g., See Lin 2007)

$$\frac{\overline{D}\,\overline{u}}{Dt} = f\,\overline{v} - \frac{1}{\rho_o}\frac{\partial \overline{p}}{\partial x} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o \overline{u'u'}\right)}{\partial x} + \frac{\partial\left(\rho_o \overline{u'v'}\right)}{\partial y} + \frac{\partial\left(\rho_o \overline{u'w'}\right)}{\partial z}\right] + \nu\nabla^2 \overline{u}, \qquad (14.1.3)$$

$$\frac{\overline{D}\,\overline{v}}{Dt} = -f\,\overline{u} - \frac{1}{\rho_o}\frac{\partial \overline{p}}{\partial y} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o \overline{u'v'}\right)}{\partial x} + \frac{\partial\left(\rho_o \overline{v'v'}\right)}{\partial y} + \frac{\partial\left(\rho_o \overline{v'w'}\right)}{\partial z}\right] + \nu\nabla^2 \overline{v}, \qquad (14.1.4)$$

$$\frac{\overline{D}\,\overline{w}}{Dt} = -\frac{1}{\rho_o}\frac{\partial p_1}{\partial z} - g\frac{\rho_1}{\rho_o} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o \overline{u'w'}\right)}{\partial x} + \frac{\partial\left(\rho_o \overline{v'w'}\right)}{\partial y} + \frac{\partial\left(\rho_o \overline{w'w'}\right)}{\partial z}\right] + \nu\nabla^2 \overline{w}, \qquad (14.1.5)$$

$$\frac{\bar{D}\bar{\theta}}{Dt} = \overline{S_\theta} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o\overline{u'\theta'}\right)}{\partial x} + \frac{\partial\left(\rho_o\overline{v'\theta'}\right)}{\partial y} + \frac{\partial\left(\rho_o\overline{w'\theta'}\right)}{\partial z}\right] + \kappa\nabla^2\bar{\theta}, \tag{14.1.6}$$

$$\frac{\bar{D}\bar{\phi}}{Dt} = \overline{S_\phi} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o\overline{u'\phi'}\right)}{\partial x} + \frac{\partial\left(\rho_o\overline{v'\phi'}\right)}{\partial y} + \frac{\partial\left(\rho_o\overline{w'\phi'}\right)}{\partial z}\right] + \kappa\nabla^2\bar{\phi},$$

$$\phi = q_v, q_c, q_i, q_r, q_s, q_g, \tag{14.1.7}$$

$$\nabla\cdot\left(\rho_o\bar{\mathbf{V}}\right) = 0, \qquad \bar{V} = (u, v, w), \tag{14.1.8}$$

$$\bar{p} = \bar{\rho}R_d\bar{T}, \tag{14.1.9}$$

$$\bar{\theta} = \overline{T_v}\left(\frac{p_{oo}}{\bar{p}}\right)^{R_d/c_p}, \tag{14.1.10}$$

$$\overline{T_v} = \bar{T}(1 + 0.61\overline{q_v}), \tag{14.1.11}$$

$$\bar{p} = p_o + p_1; \qquad \bar{\rho} = \rho_o + \rho_1; \quad \frac{\partial p_o}{\partial z} = -\rho_o g, \tag{14.1.12}$$

where

$$\frac{\bar{D}}{Dt} = \frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x} + \bar{v}\frac{\partial}{\partial y} + \bar{w}\frac{\partial}{\partial z},$$

$$\phi_o \equiv \frac{1}{Dx\,Dy}\int_y^{y+Dy}\int_x^{x+Dx} \bar{\phi}\;dx\,dy.$$

- In the above, $\overline{v'\theta'}$, and $\overline{w'\theta'}$ are turbulent heat fluxes, $\overline{u'w'}$ and $\overline{v'w'}$ are vertical turbulent fluxes of zonal momentum, and $\overline{u'v'}$ is the horizontal turbulent flux of zonal momentum.

- In order to "close" the system (closure problem), the flux terms need to be represented (parameterized) by the grid-volume averaged terms (terms with "upper bar"s).

- Different averaging methods

  Time averaging: a variable ϕ may be employed for a sensor located at a certain location (x_o, y_o, z_o),

$$\overline{\phi_t} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \phi(x_o, y_o, z_o, t) dt. \qquad (14.1.13)$$

  Space averaging:

$$\overline{\phi_s} = \lim_{X,Y,Z \to \infty} \frac{1}{XYZ} \int_{-Z/2}^{Z/2} \int_{-Y/2}^{Y/2} \int_{-X/2}^{X/2} \phi(x, y, z, t_o)\, dx\, dy\, dz. \qquad (14.1.14)$$

  Ensemble averaging: (for a data set measured discretely)

$$\overline{\phi_e} = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} \phi(x_o, y_o, z_o, t_o). \qquad (14.1.15)$$

  Grid-volume averaging: defined in (14.1.2).

  In case there exist N data points to be averaged over a grid volume, one may take the *generalized ensemble averaging*,

$$\overline{\phi} = \frac{1}{TXYZ} \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} \int_{-Z/2}^{Z/2} \int_{-Y/2}^{Y/2} \int_{-X/2}^{X/2} \int_{-T/2}^{T/2} \phi(x', y', z', t')\, dt'\, dx'\, dy'\, dz'. \qquad (14.1.16)$$

14.2 Characteristics and Parameterization of Planetary Boundary Layer (PBL) Processes

- Slab model: The PBL is treated as one slab and only the vertically averaged properties are predicted.

  Some general circulation models (*GCM*) take this approach. It is not suitable for mesoscale and NWP models.

- DNS and FTS models: Great, but too expensive.
- LES models: Great choice, maybe likely in the future.
- Convective Boundary Layer Structure

  (1) *Viscous sublayer*: molecular motions dominate the transfer of dependent variables.

  The turbulent flux terms are negligible, while the molecular viscosity and thermal diffusion terms in these equations should be kept.
  (2) *Roughness layer* or *canopy layer*: the lower part of the surface layer.

  The roughness layer may go up to 10 m over large buildings (Oke 1978). Due to the constraint of vertical resolution, the viscous sublayer and roughness layer is often neglected by mesocale models.

  (3) Above the viscous sublayer or roughness layer exists the *surface layer* (10 or 100 m, about 10% of the entire PBL).

The surface layer is mainly maintained by the vertical momentum transfer associated with turbulent eddies. Coriolis and pressure gradient forces do not play a major role in the surface layer.

(4) The layer above the surface layer is called *mixed layer* (unstable or convective) or *outer layer* (neutral or stable).

Fig. 14.2.1

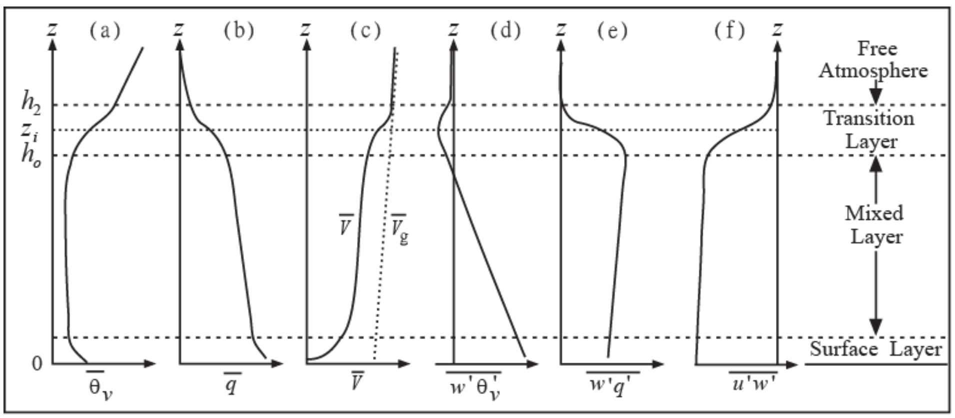

Fig. 14.2.1: Typical convective boundary layer profiles of mean virtual potential temperature, specific humidity, wind speed, vertical heat flux, vertical moisture flux, and vertical momentum flux. (Adapted from Driedonks and Tennekes, 1984)

(5) On top of the mixed layer is the *transition layer*
It contains a temperature inversion and an increase in wind speed.

14.2.1 Modeling the Surface Layer

K theory:

The subgrid scale fluxes may be represented by

$$\overline{u'w'} = -K_m \frac{\partial \bar{u}}{\partial z}; \quad \overline{v'w'} = -K_m \frac{\partial \bar{v}}{\partial z}; \quad \overline{w'\theta'} = -K_h \frac{\partial \bar{\theta}}{\partial z}; \quad \overline{w'q'} = -K_q \frac{\partial \bar{q}}{\partial z}, \quad (14.2.1)$$

where K_m is called the *exchange coefficient of momentum* or simply *eddy viscosity*, and K_h, and K_q are called the *exchange coefficients* or *eddy diffusivities of heat and water vapor*, respectively.

14.2.2 Modeling the PBL above the Surface Layer

a. Bulk Aerodynamic Parameterization

The boundary layer is treated as a single slab and assume the wind speed and potential temperature are independent of height, and the turbulence is horizontally homogeneous.

$$\overline{u'w'} = -C_d \bar{V}^2 \cos\mu; \quad \overline{v'w'} = -C_d \bar{V}^2 \sin\mu; \quad \overline{w'\theta'} = -C_h \bar{V}^2 \left[\bar{\theta} - \bar{\theta}_{z_o}\right], \quad (14.2.15)$$

where C_d and C_h are nondimensional *drag and heat transfer coefficients*, respectively,

b. K-theory parameterization

In this approach, the turbulent flux terms in (14.1.3)-(14.1.7) are written as,

$$\overline{u'w'} = -K_m \frac{\partial \bar{u}}{\partial z}; \quad \overline{v'w'} = -K_m \frac{\partial \bar{v}}{\partial z}; \quad \overline{w'\theta'} = -K_h \frac{\partial \bar{\theta}}{\partial z}; \quad \overline{w'q'} = -K_q \frac{\partial \bar{q}}{\partial z}. \quad (14.2.1)$$

If the gradient terms of (14.2.1) (e.g., $\partial \bar{u} / \partial z$) are calculated based on local gradients, it is call <u>local closure</u>; otherwise it is called <u>non-local closure</u>. Normally, a non-local closure would do a better job for a convective boundary layer.

c. Turbulent Kinetic Energy (TKE or 1 1/2) closure scheme

The TKE, $(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})/2$, is predicted, while the other subgrid scale turbulent flux terms are diagnosed and related to the TKE and to the grid-scale mean values.

$$\frac{\partial \bar{e}}{\partial t} = -\bar{\boldsymbol{V}} \cdot \nabla \bar{e} - \overline{\boldsymbol{V}' \cdot \nabla e} - (1/\rho_o)\left[(\overline{u'p'})_x + (\overline{v'p'})_y + (\overline{w'p'})_z\right] - (g/\rho_o)\overline{\rho' w'}$$

1 2 3 4

$$-\left[\left(\overline{u'u'}\,\bar{u}_x + \overline{u'v'}\,\bar{u}_y + \overline{u'w'}\,\bar{u}_z\right) + \left(\overline{u'v'}\,\bar{v}_x + \overline{v'v'}\,\bar{v}_y + \overline{v'w'}\,\bar{v}_z\right)\right. \quad (14.2.31)$$

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$$\left. + \left(\overline{u'w'}\,\bar{w}_x + \overline{v'w'}\,\bar{w}_y + \overline{w'w'}\,\bar{w}_y\right)\right] + \nu \nabla^2 \bar{e} - \nu\left(\overline{u'^2_x} + \overline{v'^2_y} + \overline{w'^2_z}\right)$$

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d. Higher-order closure schemes

In fact, the subgrid-scale perturbations, such as u', v', w', θ', can be predicted by subtracting the resolved flow equations from the full equations, similar to the derivation of TKE equation.

This will generate new unknowns involving triple correlation of the perturbations, which must be represented by the mean variables and quadratic perturbation terms, in order to close the system mathematically. This is referred to as the *second-order closure*.