Chapter 8 Quasi-Geostrophic (QG) Prediction of Geopotential Tendency

8.1 Geopotential Tendency Equation

Purpose: To derive a prognostic equation for predicting geopotential tendency.

➢ Based on the hydrostatic equation

$$\frac{\partial \phi}{\partial p} = -\frac{RT}{p}$$
(6.2)

We have

$$T = -\frac{p}{R}\frac{\partial \phi}{\partial p}$$

Substituting it into the QG thermodynamic equation

$$\frac{\partial T}{\partial t} = -u_g \frac{\partial T}{\partial x} - v_g \frac{\partial T}{\partial y} + \left(\frac{\sigma p}{R}\right)\omega + \frac{J}{c_p}$$
(6.13)

leads to

$$\frac{\partial \chi}{\partial p} = -V_g \cdot \nabla \frac{\partial \phi}{\partial p} - \sigma \omega - \frac{\kappa J}{p}$$
(6.22)

where $\chi \equiv \partial \phi / \partial t$ (geopotential tendency) and $\kappa = R/c_p$.

➢ Equation (6.22) is also called "hydrostatic thermodynamic equation".

Q: What is the physical meaning of individual terms of (6.22)?

➢ Equations (6.22) and the QG vorticity equation (6.18)'

$$\frac{\partial \zeta_g}{\partial t} + u_g \frac{\partial \zeta_g}{\partial x} + v_g \frac{\partial \zeta_g}{\partial y} = f_0 \frac{\partial \omega}{\partial p} - \beta v_g \qquad (6.18)$$

or

$$\frac{1}{f_0}\nabla^2\chi + u_g \frac{\partial}{\partial x}\left(\frac{1}{f_0}\nabla^2\phi\right) + v_g \frac{\partial}{\partial y}\left(\frac{1}{f_0}\nabla^2\phi\right) = f_0 \frac{\partial \omega}{\partial p} - \beta v_g \qquad (6.18)'$$

form a closed set of equations of ϕ and ω since

$$\zeta_g = \frac{1}{f_0}\nabla_p^2\phi \qquad (6.13)$$

➢ Eliminate ω ⟹ geopotential tendency (χ) equation
⟹ To predict geopotential height tendency
➢ Eliminate χ ⟹ Omega (ω) equation
⟹ To diagnose vertical motion

The geopotential tendency equation can then be derived

$$\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_o^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\chi = -f_o V_g.\nabla(\frac{1}{f_o}\nabla^2\phi + f) - \frac{\partial}{\partial p}\left[\frac{-f_o^2}{\sigma}V_g \bullet \nabla\left(-\frac{\partial\phi}{\partial p}\right)\right] - (f_o^2\kappa)\frac{\partial}{\partial p}\left(\frac{J}{\sigma}\right)$$

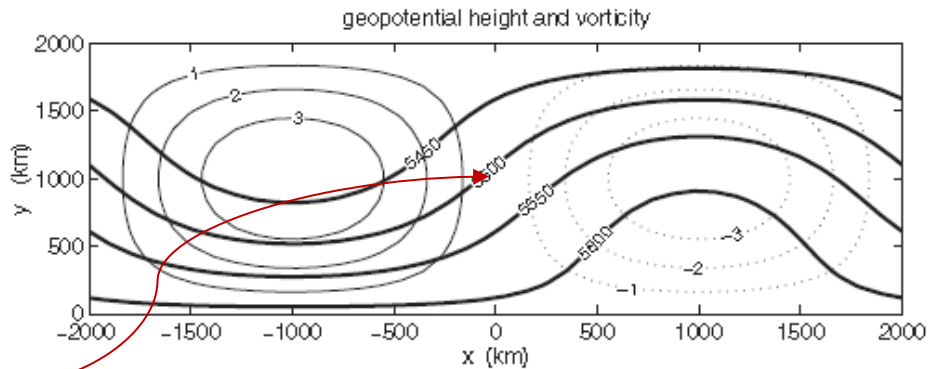Term A   Term B   Term C   Term D   (6.23)

Physical meaning of (6.23) may be understood by the following simple form:

$$-\chi \propto -V_g \cdot \nabla(\zeta_g + f) + \frac{\partial}{\partial z}(-V_g \cdot \nabla T)$$

or

$$-\chi \propto -V_g \cdot \nabla\zeta_g - \beta v_g + \frac{\partial}{\partial z}(-V_g \cdot \nabla T)$$

Term B: (1) Relative Vorticity Advection $(-V_g \cdot \nabla\zeta_g)$

geopotential height and vorticity

At $(x_o, y_o) = (0, 1000km)$
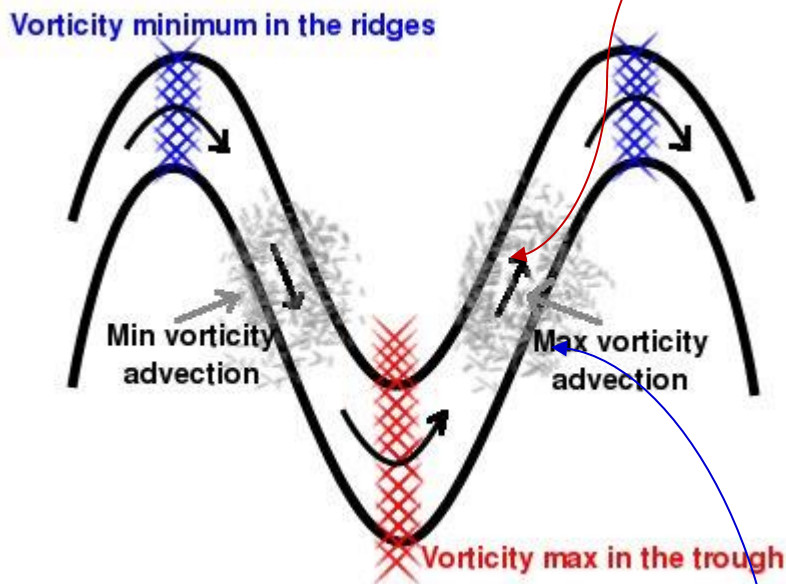
$$-\chi \propto -V_g \cdot \nabla\zeta_g \propto -U_g(\frac{\partial\zeta_g}{\partial x}) > 0$$

$$+ \quad -$$

$$-\chi = -\frac{\partial\phi}{\partial t} > 0$$

$$\frac{\partial\phi}{\partial t} < 0 \quad => \quad \phi\,\text{(height) decreases with time at } (x_o, y_o)$$

The wave is propagating eastward. [Trough will move to here!]

Vorticity minimum in the ridges

Min vorticity advection

Max vorticity advection

Vorticity max in the trough

(2) Planetary Vorticity Advection $(-\beta v_g)$

$$-\chi \propto -\beta v_g = -v_g \frac{\partial f}{\partial y}$$

At (x_o, y_o):   +   +

$$-\chi = -\frac{\partial \phi}{\partial t} < 0$$

ϕ (height) increases with time at (x_o, y_o) [Ridge will move to here!]

The wave is propagating westward (retrogressive).

Term C: Differential Temperature Advection $[\frac{\partial}{\partial z}(-V_g \cdot \nabla T)]$
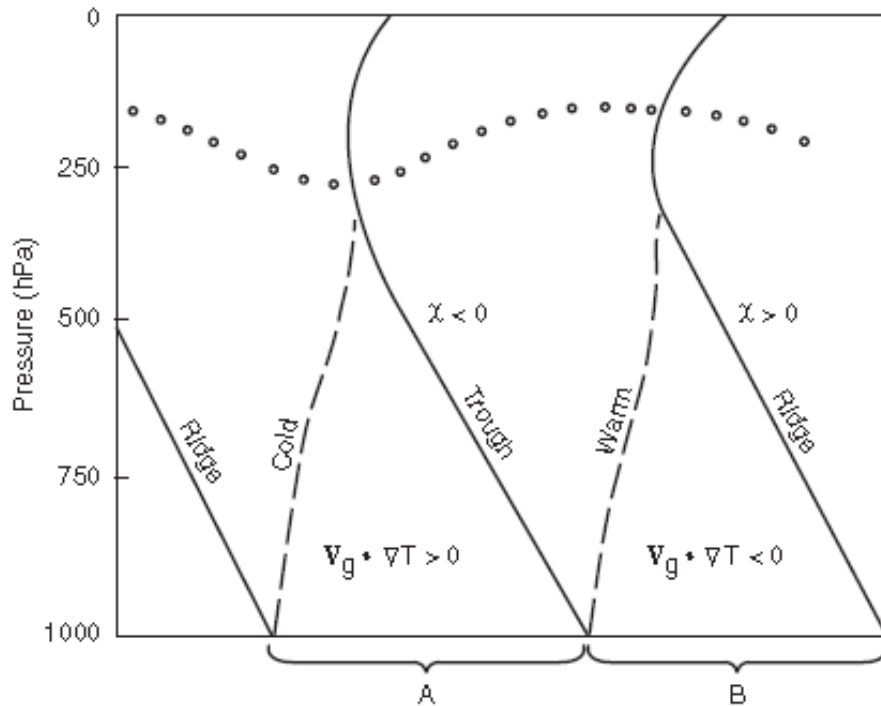

Fig. 6.9 East–west section through a developing synoptic disturbance showing the relationship of temperature advection to the upper level height tendencies. A and B designate, respectively, regions of cold advection and warm advection in the lower troposphere.

$$-\chi \propto \frac{\partial(-V_g \cdot \nabla T)}{\partial z} \approx \frac{(-V_g \cdot \nabla T)_2 - (-V_g \cdot \nabla T)_1}{\Delta z} \approx \frac{(V_g \cdot \nabla T)_1}{\Delta z}$$

(because the temperature advection is dominated in lower layer)

Thus in area A: $-\chi = -\frac{\partial \phi}{\partial t} < 0 \implies$

ϕ (height) increases with time [Ridge will move to here!]

In area B: ϕ (height) decreases with time [Trough will move to here!]

5