## Lecture 6 Heat

(Ch. 3 First Law of Thermodynamics
(3.2a Heat)

Consider the following system:

Initial State $\left(T_{1}, T_{2}\right)$


Warm Body Cold Body

$$
T_{1}>T_{2}
$$



$$
T_{1}>T_{F}>T_{2}
$$

The energy which exchanged from the warm body to the cold body is called heat.

The amount of heat loss by the warm body is proportional to the temperature difference between the initial and the final states of that body:

$$
\begin{equation*}
-\Delta Q=C_{1}\left(T_{F}-T_{1}\right) \tag{3.5}
\end{equation*}
$$

Similarly, the amount of heat gained by the cold body is proportional to $T_{F}-T_{2}$,

$$
\begin{equation*}
\Delta Q=C_{2}\left(T_{F}-T_{2}\right) \tag{3.6}
\end{equation*}
$$

where $C_{1}$ and $C_{2}$ are called heat capacities of the two these bodies.

The amount of heat gained by the cold body must be the same as the amount of heat lost by the warm body. Adding (3.5) and (3.6) gives

$$
\begin{equation*}
C_{1}\left(T_{F}-T_{1}\right)+C_{2}\left(T_{F}-T_{2}\right)=0 \tag{3.7}
\end{equation*}
$$

From Eq. (3.5), we have

$$
C_{1}=\frac{-\Delta Q}{\left(T_{F}-T_{1}\right)}=\frac{-\Delta Q}{-\left(T_{1}-T_{F}\right)}=\frac{\Delta Q}{\Delta T_{1}}
$$

where $\Delta T_{1}=T_{1}-T_{F}$. Thus, $C_{1}$ is the heat loss by body 1 (warm body) lost per degree K.

Similarly, $\mathrm{C}_{2}$ can be derived from Eq. (3.6),

$$
C_{2}=\frac{\Delta Q}{\left(T_{F}-T_{2}\right)}=\frac{\Delta Q}{\Delta T_{2}}
$$

where $\Delta T_{2}=T_{F}-T_{2}$. Thus, $C_{2}$ is the heat gained by body 2 (cold body) gained per degree K .
Clearly, as $\Delta T_{1}$ and $\Delta T_{2}$ approach $0, T_{1}, T_{2}$ and $T_{F}$ approach to the same value. In other words, we have

$$
C_{1}=\lim _{\Delta T_{1} \rightarrow 0} \frac{\Delta Q}{\Delta T_{1}}=\lim _{\Delta T_{2} \rightarrow 0} \frac{\Delta Q}{\Delta T_{2}}=C_{2}=\left.\frac{d Q}{d T}\right|_{T=T_{F}}
$$

Thus, we can define the heat capacity at temperature $T$ as,

$$
\begin{equation*}
C=\frac{d Q}{d T} \tag{3.8}
\end{equation*}
$$

Divide (3.8) by mass $m$, we get the expression for specific heat capacity,

$$
\begin{equation*}
c=\frac{d q}{d T} \tag{3.9}
\end{equation*}
$$

where $q=Q / m, c=C / m$. The unit for the specific heat is $J K^{-1} \mathrm{~kg}^{-1}$.
If the transformation process from $T_{1}$ to $T_{F}$ is isobaric, the specific heat is called the specific heat at constant pressure:

$$
c_{p}=\left.\left(\frac{d q}{d T}\right)\right|_{p}
$$

Similarly, if the transformation process is isosteric, the specific heat is called the specific heat at constant volume:

$$
c_{v}=\left.\left(\frac{d q}{d T}\right)\right|_{v}
$$

For example,
For water vapor, $c_{v}=1463 \mathrm{Jkg}^{-1} \mathrm{~K}^{-1}, c_{p}=1952 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$. For dry air, $\quad c_{\nu}=717 \mathrm{Jkg}^{-1} \mathrm{~K}^{-1}, c_{p}=1004 \mathrm{Jkg}^{-1} \mathrm{~K}^{-1}$.

Note that:
(a) In general $c_{p}>c_{v}$.

This may be explained by considering the piston-cylinder system. The amount of heat needed to heat the air in the cylinder with a constant pressure $\left(c_{p}\right)$ is larger than that with a constant pressure $\left(c_{v}\right)$ because part of the heat is used to do work of expansion on the environment.
(b) $c_{p}$ and $c_{v}$ are temperature dependent Strictly speaking, an exact value of $c_{p}$ or $c_{v}$ only applies to the condition of a certain temperature. However, for gas (e.g., air and water vapor), the difference for different temperatures is small. Thus, normally we just use the constant values as shown above.
(c) Heat has a unit of Joule ( Nm or $\mathrm{kgm}^{2} \mathrm{~s}^{-2}$ )

It is the same as work and energy. Sometimes heat is measured by calorie, which is defined as the quantity of heat needed to raise the temperature of 1 gram of pure water from $14.5{ }^{\circ} \mathrm{C}$ to $15.5^{\circ} \mathrm{C}$. The relation between $J$ and calorie is 1 calorie $=4.186$ $J$.

