7.4 QG Diagnosis: Vertical Motion

Diagnoze vertical motion in the atmosphere:

Our Challenge:

- We do not observe vertical motion
- Intimately linked to clouds and precipitation
- Actual vertical motions are often several orders of magnitude smaller than their collocated horizontal air motions [w ~ 0.01 → 10 m/s] [u,v ~ 10 → 100 m/s]
- Synoptic-scale vertical motions must be estimated from widely-spaced observations (i.e. the rawindsonde network) every 12-hours

Methods:

- Kinematic Method — Integrate the Continuity Equation. Very sensitive to small errors in winds measurements

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \omega}{\partial p}=0,$$

to estimate ω at p,

$$\omega(p)=\omega(p_s)+(p_s-p)\left[\frac{\partial\langle u\rangle}{\partial x}+\frac{\partial\langle v\rangle}{\partial y}\right]_p . \qquad \text{H(3.38)}$$

QG Diagnosis: Vertical Motion

- Adiabatic Method — From the thermodynamic equation
  Very sensitive to temperature tendencies (too coarse)
  Difficult to incorporate impacts of diabatic heating

$$\omega = \frac{1}{S_p}\left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right]. \qquad \text{H(3.41)}$$

- QG Omega Equation — Least sensitive to small observational errors
  Widely believed to be the best method

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The Quasigeostrophic Omega Equation:

- We can also derive a ***single*** diagnostic equation for ω by, again, combining our vorticity and thermodynamic equations (the height-tendency versions from before):

$$\boxed{\frac{1}{f_0}\nabla_p^2\chi + u_g\frac{\partial}{\partial x}\left(\frac{1}{f_0}\nabla_p^2\phi\right) + v_g\frac{\partial}{\partial y}\left(\frac{1}{f_0}\nabla_p^2\phi\right) = f_0\frac{\partial\omega}{\partial p} - \beta v_g}$$

ω

$$\boxed{\frac{\partial\chi}{\partial p} + u_g\frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial p}\right) + v_g\frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial p}\right) = -\sigma\omega}$$

- To do this, we need to eliminate the height tendency (χ) from both equations

Step 1: Apply the operator $f_0\frac{\partial}{\partial p}$ to the vorticity equation

Step 2: Apply the operator ∇_p^2 to the thermodynamic equation

Step 3: Subtract the result of Step 1 from the result of Step 2

A diagnostic equation of ω can the be derived.

The Quasigeostrophic Omega Equation:

$$\boxed{\underbrace{\left(\nabla_p^2 + \frac{f_0^{\,2}}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} \;=\; \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-V_{\mathbf{g}}\cdot\nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} \;\underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-V_{\mathbf{g}}\cdot\nabla_p T\right)}_{\text{Term C}}}$$

• The above equation can be simplified to be

$$\boxed{w \;=\; \frac{\partial}{\partial z}\left[-V_{\mathbf{g}}\cdot\nabla_p\left(\zeta_g + f\right)\right] \;-V_{\mathbf{g}}\cdot\nabla_p T}$$

Term A: Local Vertical Motion

QG Diagnosis: Vertical Motion

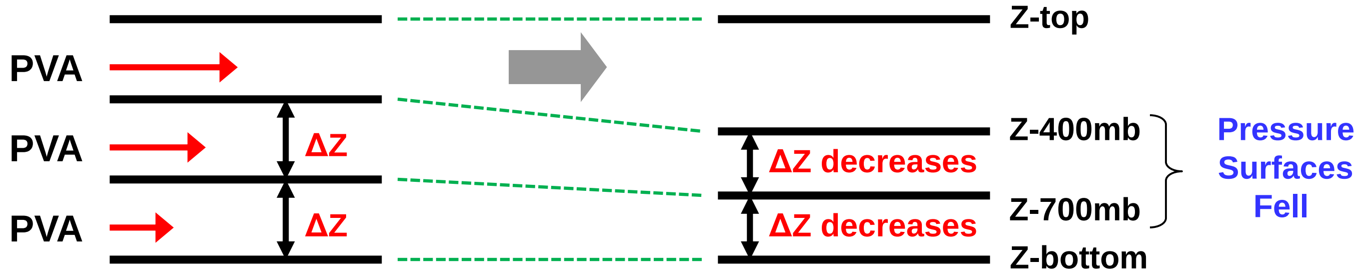
Further Explanation of The QG Omega Equation (Term B):

$$w = \frac{\partial}{\partial Z}\left[-V_{\mathbf{g}} \cdot \nabla_p\left(\zeta_g + f\right)\right] \quad -V_{\mathbf{g}} \cdot \nabla_p T$$

Term A **Term B** **Term C**

Term B: Change in Absolute Vorticity Advection with "Height"

- Recall, positive (relative) vorticity advection (**PVA**) leads to local height falls
- Consider a three-layer atmosphere where cyclonic vorticity advection increases with height, or **PVA** is strongest in the upper layer:



- Hydrostatic balance (and the hypsometric equation) requires ALL changes in thickness (ΔZ) to be accompanied by temperature changes…

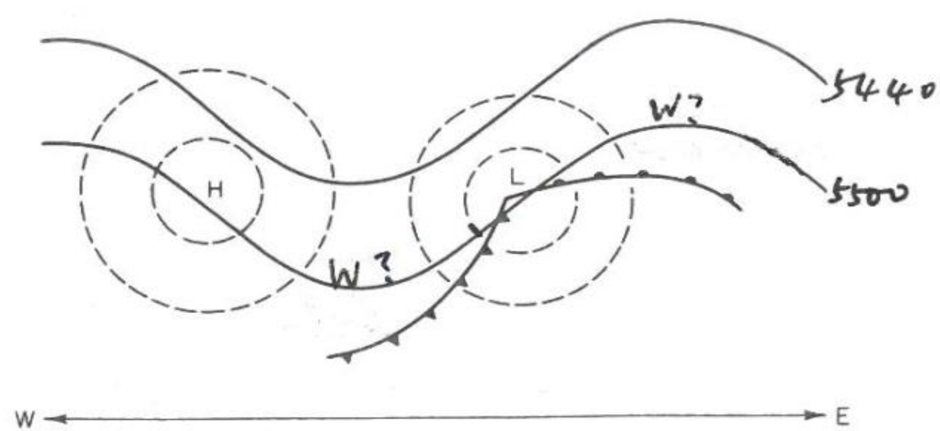
The Quasigeostrophic Omega Equation:

$$\boxed{\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{V_g}\cdot\nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{V_g}\cdot\nabla_p T\right)}_{\text{Term C}}}$$

• The above equation can be simplified to be

$$\boxed{w = \frac{\partial}{\partial z}\left[-\mathbf{V_g}\cdot\nabla_p\left(\zeta_g + f\right)\right] - \mathbf{V_g}\cdot\nabla_p T}$$

Term B: Differential Advection

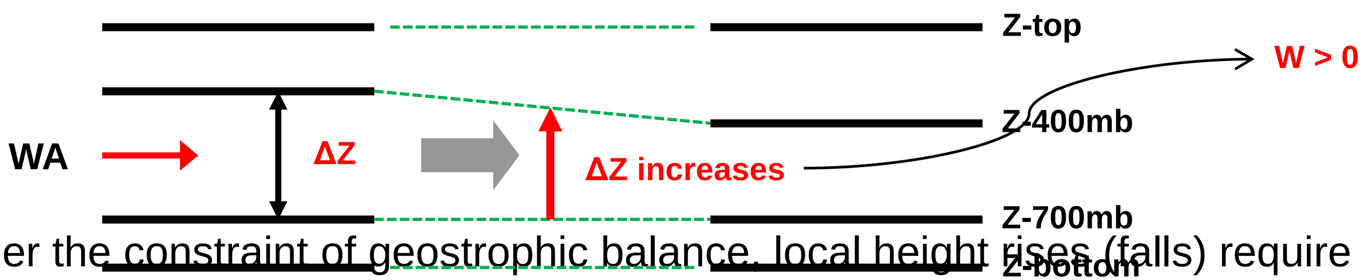


Further Explanation of The QG Omega Equation (Term C):

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v_g}\bullet\nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v_g}\bullet\nabla_p T\right)}_{\text{Term C}}$$

Term C: Horizontal Temperature Advection

- Warm air advection (**WA**) leads to local temperature increases
- Consider the three-layer model, with **WA** strongest in the middle layer



- Under the constraint of geostrophic balance, local height rises (falls) require a change in the local pressure gradient, a change in the local geostrophic wind, and thus a local decrease (increase) in <u>geostrophic</u> vorticity…..

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