

Project 3

(2/8/21; Due 2/15/21)

[Students are encouraged to think creatively and perform extra experiments if necessary, based on the simple experiments given by the projects. Extra credits will be given for more thorough and creative results.]

1.

(a) Look for an analytical solution in form of $u'(t, x) = u_i(x-ct)$ for the one-dimensional (1D) advection equation of Project 2,

$$\frac{\partial u'}{\partial t} + U \frac{\partial u'}{\partial x} = 0,$$

where U is a constant. What are the physical meanings of u_i and c in the advection equation? If u_i is a bell-shaped function with amplitude u_o and half-width b , what is the analytical solution $u'(t, x)$?

(b) Using the linear result from Project 2 to estimate the amplitude and advection speed of the wave, and compare them with the u_o and c from the analytical linear solution. Try your best to explain the results. In your discussion, you need to insert your figures (cut, paste, and shrink) into the text of discussion.

2.

(a) Run the advection model for the nonlinear advection equation (inviscid Burger Equation) (i.e., set $NL = 1$):

$$\frac{\partial u'}{\partial t} + (U + u') \frac{\partial u'}{\partial x} = 0.$$

(b) Make a number of sensitivity tests by increasing Δt in the Advection Model (again, let $NL=1$) to identify the maximum time interval (Δt), which gives a well-behaved solution, i.e. numerically stable. You may consider reducing the Δt more significantly, if you do not see much different result from the previous run. Construct a table to show the maximum amplitude of u' versus the time interval for each case you have run. Make a couple of plots of $u'(x, t)$ versus x at a time when the numerical solution starts to become unstable.