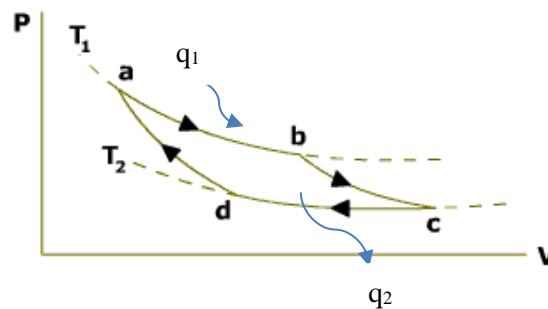


## Lecture 12 The 2nd Law of Thermodynamics

(Sec.3.6 of Hess – 2<sup>nd</sup> Law of Thermodynamics)

➤ **Claim:** In a Carnot cycle,  $q_2 / q_1 = T_2 / T_1$ .



**Proof:**

Applying the first law of thermodynamics ( $dq = c_v dT + p d\alpha$ ) to the isothermal processes  $a \Rightarrow b$  and  $c \Rightarrow d$  leads to

$$q_1 = \int_a^b p d\alpha = \int_a^b \frac{RT_1}{\alpha} d\alpha = RT_1 \ln \frac{\alpha_b}{\alpha_a} \quad (12.1)$$

$$q_2 = - \int_c^d p d\alpha = - \int_c^d \frac{RT_2}{\alpha} d\alpha = RT_2 \ln \frac{\alpha_c}{\alpha_d} \quad (12.2)$$

Equations (12.1) and (12.2) gives

$$\frac{q_2}{q_1} = \frac{RT_2 \ln(\alpha_c/\alpha_d)}{RT_1 \ln(\alpha_b/\alpha_a)} = \left( \frac{T_2}{T_1} \right) \frac{\ln(\alpha_c/\alpha_d)}{\ln(\alpha_b/\alpha_a)}$$

Therefore, in order to have  $q_2 / q_1 = T_2 / T_1$ , it requires

$$\frac{\alpha_c}{\alpha_d} = \frac{\alpha_b}{\alpha_a} \quad (12.3)$$

Applying the equation of state,

$$p\alpha = RT$$

to the initial and final states of the isothermal processes  $a \Rightarrow b$  and  $d \Rightarrow a$  and the Poisson's equation,

$$p\alpha^\gamma = \text{constant}$$

to the adiabatic processes  $b \Rightarrow c$  and  $d \Rightarrow a$  gives

$$p_a\alpha_a = p_b\alpha_b = RT_1 \tag{1}$$

$$p_b\alpha_b^\gamma = p_c\alpha_c^\gamma \tag{2}$$

$$p_c\alpha_c = p_d\alpha_d = RT_2 \tag{3}$$

$$p_d\alpha_d^\gamma = p_a\alpha_a^\gamma. \tag{4}$$

Eq. (3) gives

$$\frac{\alpha_c}{\alpha_d} = \frac{p_d}{p_c} \tag{5}$$

Eqs. (2) and (4) gives

$$P_c = P_b(\alpha_b^\gamma/\alpha_c^\gamma)$$

$$P_d = P_a(\alpha_a^\gamma/\alpha_d^\gamma)$$

Substituting  $p_c$  and  $p_d$  into (5) yields

$$\frac{\alpha_c}{\alpha_d} = \left(\frac{p_a}{p_b}\right) \left(\frac{\alpha_a^\gamma\alpha_c^\gamma}{\alpha_b^\gamma\alpha_d^\gamma}\right). \tag{7}$$

Eq. (1) gives

$$p_a/p_b = \alpha_b/\alpha_a \quad (7)$$

Substituting (7) into (6) leads to

$$\frac{\alpha_c}{\alpha_d} = \frac{\alpha_b}{\alpha_a}, \quad (12.3)$$

which proves  $q_2/q_1 = T_2/T_1$ .

### ➤ *The Second Law of Thermodynamics*

The above claim leads to the 2<sup>nd</sup> Law of Thermodynamics, which concerns about the maximum fraction of a quantity of heat that can be converted into useful work.

From the expression of the efficiency of heat engine:

$$\eta = 1 - \frac{q_2}{q_1} = 1 - \frac{T_2}{T_1}$$

Therefore, in order to have a 100% efficiency ( $\eta = 1$ ),  $T_2$  must be zero ( $T_2 = 0 \text{ K}$ ). Is it possible? The answer is “no”.

#### The 2nd law of thermodynamics (Kelvin-Planck statement):

It is impossible to construct an engine which operates in a cycle and which produces no other effect than the extraction of heat from a heat reservoir and the performance of an equivalent amount of work.

#### Alternative statement:

Heat cannot of itself (i.e., without the performance of work by some external agency) pass from a cold to a warm body.