Department of Physics Dr. Yuh-Lang Lin
http://mesolab.org ylin@ncat.edu

Lecture 11 Carnot Cycle

(Sec.3.6 of Hess – 2nd Law of Thermodynamics)
[For classical equation editor: ($dq = 0$)]

- Thermodynamic processes can be divided into three different categories:
  (1) reversible processes
  (2) non-existing (impossible) processes
  (3) real (natural) processes

- (1) Reversible processes: If each state of the system is in equilibrium so that a reversal in the direction of an infinitesimal change returns the system and environment to their original states (determined by p, α, T). For example:
  Carnot cycle, which will be discussed later in this lecture, is an example of reversible processes.

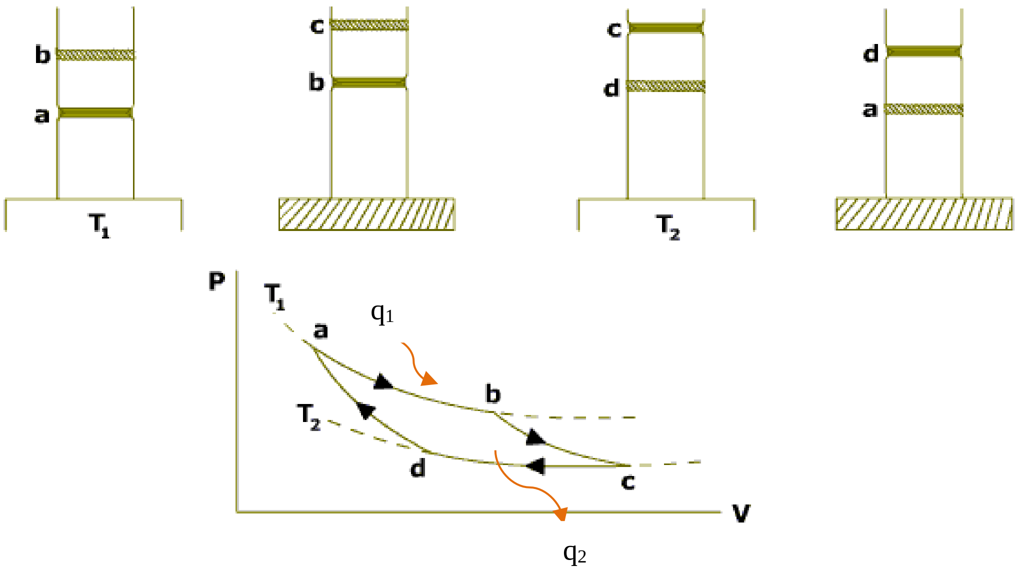
- (2) Non-existing or impossible processes
  For example:
  (a) Compression of a gas with no external pressure
  (b) Let the heat flow from a cold body to a warm body without doing work.

- (3) Natural processes are always more or less irreversible
  For example:
  (a) Heat conduction through a finite temperature gradient
  (b) Combination of oxygen and hydrogen at room temperature into water
  (c) Diffusion of one gas into another
  (d) Cloud formation

12.1 The Carnot Cycle

- Carnot cycle: Carnot cycle refers to thermodynamic processes occurring in Carnot's ideal heat engine.

- Carnot's engine and cycle: it consists the following cyclic processes in a P-V (or α-p) diagram.



(1) a => b: isothermal expansion
The cylinder is placed on a hot reservoir, which an amount of heat q_1 added into the cylinder with the temperature kept constant (T_1).
Equation of state: $dq = pd\alpha$, $dq = q_1$.

(2) b => c: adiabatic expansion
The cylinder is placed on an insulated stand, which keeps expanding, but the temperature falls from T_1 to T_2 ($T_2 < T_1$) since there is no heat added (adiabatic). Equation of state: $0 = c_v dT + pd\alpha$, $dT = T_2 - T_1 < 0$.

(3) c => d: isothermal compression
The cylinder is placed on stand a cold reservoir, which extracts an amount of heat q_2 from the cylinder with the temperature kept constant at T_2.
Equation of state: $dq = pd\alpha$, $dq = -q_2$.

(4) d => a: adiabatic compression
The cylinder is placed on stand an insulated stand, which keeps compressing, but the temperature rises from T_2 to T_1 ($T_1 > T_2$) due to

compression and adiabatic process. Equation of state: $0 = c_v dT + pd\alpha$, $dT = T_1 - T_2 > 0$.

This video may help you to understand the Carnot cycle.

All the above processes are reversible and in equilibrium, which can be illustrated in a P-V (or α-p) diagram as shown above.

The net work done during the Carnot cycle is the area enclosed by ABCD, i.e.,

$$w = \oint pd\alpha = \oint dq - \oint c_v dT = q_1 - q_2$$

i.e., in this cyclic operation the engine has done work by transferring a certain amount of heat from a warmer body (H) to a cooler body (S).

- Efficiency of a heat engine: If during one cyclic process of an engine, a quantity of heat Q_1 is absorbed and heat Q_2 is rejected, the amount of mechanical work done by the engine is $Q_1 - Q_2$, then the efficiency of the engine (η) (only true for a cyclic process) is defined as

$$\eta = \frac{\text{Mechanical work done by the engine}}{\text{Heat absorbed by the engine}} = \frac{Q_1 - Q_2}{Q_1} \tag{4.1.1}$$

or, for a unit mass,

$$\eta = \frac{q_1 - q_2}{q_1} = 1 - \frac{q_2}{q_1}. \tag{4.1.2}$$

The efficiency of the Carnot heat engine is

$$\eta = \frac{q_1 - q_2}{q_1} = 1 - \frac{q_2}{q_1} = 1 - \frac{T_2}{T_1} \tag{4.1.3}$$

Q: Does $q_2 / q_1 = T_2 / T_1$? [Will be proved in the next lecture.]

- A steady-state, mature tropical cyclones behave like a Carnot cycle
  - Leg 1 is regarded as an isothermal expansion process
  - Leg 2 can be approximately regarded as an adiabatic expansion process
  - Leg 3 is isothermal compression process
  - Leg 4 can approximately be viewed as an adiabatic compression process

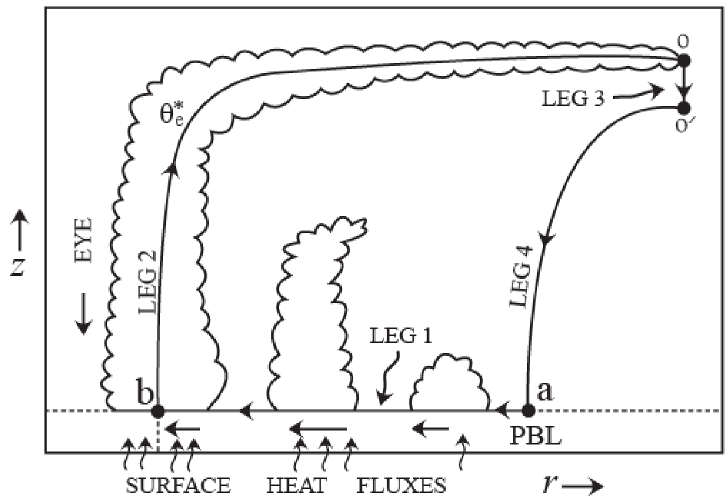


Fig. 9.19: Idealized Carnot cycle for a steady-state, mature tropical cyclone, based on the WISHE mechanism (Emanuel 1986, 1989). Solid curves represent hypothetical air trajectory in a Carnot cycle. (Lin 2007; Adapted after Emanuel 1997)